ON THE THEORY OF HYPERSONIC FLOW OF A VISCOUS GAS PAST A BLUNT BODY

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We consider the plane and axisymmetric problems of flow of a uniform hypersonic stream of viscous perfect gas past a blunt-nosed body in the case when the ordinary boundary-layer theory is inadequate, and higher approximations in the solution of the Navier-Stokes equations are required. By means of the well-known method of inner and outer expansions we obtain the conditions on the bow shock wave for the second approximation to the solution outside the boundary layer (the first approximation being the inviscid flow). We consider the boundary-value problem arising in the determination of the second approximation.

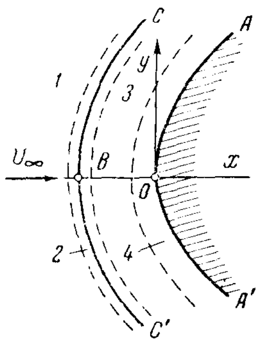

Fig. 1

In flight at high altitude with very great speed the continuum theory becomes invalid, and it is necessary to use the kinetic theory of gases. However, so long as the Knudsen number remains less than 0.15, most aerodynamic problems can be treated by means of the Navier-Stokes equations taking into account slip at the surface of the body (see the works of Street, Sherman and Talbot, and others in [1]). Under these conditions the boundary-layer theory is inadequate, and the next approximation to the solution of the Navier-Stokes equations must be considered. A systematic review of second-order effects is given in the work of Van Dyke [2], who used the well-known method of inner and outer solutions. In particular, he found the boundary conditions on the body for the determination of the second approximation outside the boundary layer. In the present work we obtain the boundary conditions on the bow shock wave for the second approximation by constructing the asymptotic representations of the solutions of the Navier-Stokes equations outside and inside the region of the bow wave and their matching.

1. We consider the plane or axisymmertic problem of hypersonic flow of a uniform stream of viscous perfect gas past a contour (Fig. 1).

Here AOA' is the contour of the body. Region 4 is the boundary layer, and region 2 the shock wave, which is regarded as a region of large gradients of gas properties. We assume that the smooth contour AOA' consists of analytic arcs, the arc of the contour being an analytic curve from point O to the limiting characteristic (see Section 6); the gas is perfect, that is, its equation of state is $p = R\rho T$ where p is the pressure, ρ the density T the absolute temperature, and R the gas constant; the specific heats at constant pressure c_p and volume c_v are constant; the internal energy is $e = c_v T$; the coefficients of viscosity μ and λ are functions only of T; the Prandtl number σ is constant; the gas flow is described by the Navier-Stokes equations; and the flow regime is laminar.

We denote the parameters of the undisturbed stream by the subscript ∞, so that M_∞ is the free-stream Mach number, and U_∞ its speed. For $\mu, \lambda \to 0$ region 2 collapses into the surface CBC', and region 4 disappears. At hypersonic speeds ($M_\infty \gg 1$) regions 2 and 4 are always distinct [2], and the characteristic temperature and enthalpy in regions 3 and 4 are $U_\infty^2 c_p^{-1}$ and U_∞^2 respectively; the radius of curvature a of the body and the radius of curvature of the shock wave at the points B and O are of the same order.

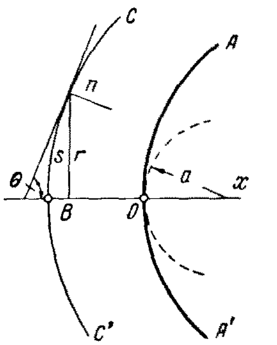

Fig. 2

We introduce a curvilinear system of coordinates s, n (Fig.2). Here s and n are measured along the arc BC and along the normal to it. Then if lengths are referred to the quantity a, the gas velocity to U_∞, the pressure to $\rho_\infty U_\infty^2$, the density to ρ_∞, the temperature to $U_\infty^2 c_p^{-1}$, the entropy and enthalpy of the gas to c_p and U_∞^2 respectively, and the coefficients of viscosity to the value of μ at $T = U_\infty^2 c_p^{-1}$, then the equations of continuity, momentum and energy, and the equations of state, assume in the chosen coordinate system the following form [2]:

$$[(r + n\cos\theta)^j \rho u]_s + [(1 + kn)(r + n\cos\theta)^j \rho v]_n = 0 \qquad (1.1)$$

$$\begin{aligned}
&\varepsilon^{-2}\left[\rho\left(u\frac{u_s}{1+kn} + vu_n + \frac{k}{1+kn}uv\right) + \frac{p_s}{1+kn}\right] = \left[\mu\left(u_n + \frac{v_s - ku}{1+kn}\right)\right]_n + \\
&+ \frac{2}{1+kn}\left[\mu\frac{u_s + kv}{1+kn}\right]_s + \mu\left(\frac{2k}{1+kn} + \frac{j\cos\theta}{r + n\cos\theta}\right)\left(u_n + \frac{v_s - ku}{1+ku}\right) + \\
&+ \frac{2j\mu}{(1+kn)(r+n\cos\theta)}\left[\frac{u_s + kv}{1+kn} - \frac{u}{(1+kn)(r+n\cos\theta)}(r + n\cos\theta)_s + \right. \\
&\left. + \frac{v\cos\theta}{r+n\cos\theta}\right](r + n\cos\theta)_s + \frac{1}{1+kn}\left[\lambda\left(\frac{u_s + kv}{1+kn} + v_n\right) + \right. \\
&\left. + \frac{j\lambda}{r+n}\left(\frac{u}{1+kn}\left(r + n\cos\theta\right)_s + v\cos\theta\right)\right]_s \qquad (1.2)
\end{aligned}$$

$$\varepsilon^{-2}\left[\rho\left(u\frac{v_s}{1+kn}+vv_n-\frac{k}{1+kn}u^2\right)+p_n\right]=2(\mu v_n)_n+$$
$$+\frac{1}{1+kn}\left[\mu\left(u_n+\frac{v_s-ku}{1+kn}\right)\right]_n+2\mu k\left(\frac{k}{1+kn}+\right.$$
$$\left.+\frac{j\cos\theta}{r+n\cos\theta}\right)v_n-2\mu\frac{k}{1+kn}\frac{u_s+kv}{1+kn}-\frac{2j\mu\cos\theta}{(r+n\cos\theta)^2}\times \tag{1.3}$$
$$\times\left[\frac{u}{1+kn}(r+n\cos\theta)_s+v\cos\theta\right]+\frac{j\mu}{(1+kn)(r+n\cos\theta)}\left(u_n+\frac{v_s-ku}{1+kn}\right)\times$$
$$\times(r+n\cos\theta)_s+\left[\lambda\frac{u_s+kv}{1+kn}+\lambda v_n+\frac{j\lambda}{r+n\cos\theta}\left(\frac{u}{1+kn}(r+n\cos\theta)_s+v\cos\theta\right)\right]_n$$

$$\varepsilon^{-2}\left[\rho\left(u\frac{T_s}{1+kn}+vT_n\right)-\left(u\frac{p_s}{1+kn}+vp_n\right)\right]=$$
$$=\frac{\sigma^{-1}}{1+kn}\left(\frac{\mu T_s}{1+kn}\right)_s+\sigma^{-1}(\mu T_n)_n+\frac{j\sigma^{-1}\mu T_s}{(1+kn)^2(r+n\cos\theta)}(r+n\cos\theta)_s+$$
$$+\sigma^{-1}\left(\frac{k}{1+kn}+\frac{j\cos\theta}{r+n\cos\theta}\right)\mu T_n+\Phi \tag{1.4}$$

$$\Phi=\mu\left[2\left(\frac{u_s+kv}{1+kn}\right)^2+2v_n^2+\frac{2j}{(r+n\cos\theta)^2}\left(\frac{n}{1+kn}(r+n\cos\theta)_s+v\cos\theta\right)^2+\right.$$
$$\left.+\left(u_n+\frac{v_s-ku}{1+kn}\right)^2\right]+$$
$$+\lambda\left[\frac{u_s+kv}{1+kn}+v_n+\frac{j}{r+n\cos\theta}+\left(\frac{u}{1+kn}(r+n\cos\theta)_s+v\cos\theta\right)\right]^2 \tag{1.5}$$

$$p=\frac{\gamma-1}{\gamma}\rho T,\qquad \varepsilon=\left[\frac{\mu(U_\infty^2c_p^{-1})}{\rho_\infty U_\infty a}\right]^{1/2},\qquad \mu=\mu(T),\qquad \lambda=\lambda(T) \tag{1.6}$$

Here u and v are the components of velocity in the directions of increasing s and n respectively; $r = r(s)$ and $\theta = \theta(s)$ are the distance from the angle of inclination of the tangent with the x-axis (Fig.2) for a point on the arc CBC', and $k = k(s)$ is its curvature, referred to a^{-1}. The subscripts s and n on a functional symbol indicate differentiation. For the plane case $j = 0$ and for the sxisymmetric case $j = 1$. For a gas with μ = const the small parameter $\varepsilon = R_\infty^{-\frac{1}{2}}$, where R_∞ is the Reynolds number of the undisturbed stream.

2. To find the solution of the problem in the boundary layer (region 4 in Fig.1), Van Dyke [2] used the method of inner and outer expansions in the parameter ε. The first terms of the inner expansion in region 4 give the usual boundary-layer theory, and the next ones account for second-order effects (in particular, slip and temperature jump at the body surface). For the flow in region 3 Van Dyke obtained asymptotic expansions of the form

$$f=F_0(s,n)+\varepsilon F_1(s,n)+\ldots\qquad (f=p,\rho,u,v,T) \tag{2.1}$$

To find the coefficients F_1 in (2.1), knowledge of which is required to improve the results of boundary-layer theory, it is necessary to give the component of velocity normal to the surface of the body, and relations between the F_1 on the bow shock wave CBC' (Fig.1). The first is available in [2] and a derivation of the conditions on the shock wave follows hereafter.

3. For the solution of this problem we use the method of inner and outer expansions. In regions 1 and 3 (Fig.1) we use for u, v, p, ρ and T expansions of the form (2.1) (the outer solution). Inside the shock wave (region 2 of Fig.1) we use expansions of the following sort (the inner solution):

$$f = f_0(s, N) + \varepsilon f_1(s, N) + \ldots, \qquad N = n\varepsilon^{-2} \tag{3.1}$$

This type of expansion is suggested by consideration of the one-dimensional case, and by the expansions (2.1). Thus let us assume that for $n \to 0$ the functions F_0, F_1, ... are representable asymptotically by power series; then for small n we obtain from (3.1)

$$f = [F_{00}(s) + F_{01}(s)\,n + \ldots] + \varepsilon\,[F_{10}(s) + F_{11}(s)\,n + \ldots] + \ldots \tag{3.2}$$

After transforming to N and regrouping terms in the series (3.2) we have

$$f = [F_{00}(s)] + \varepsilon\,[F_{10}(s)] + \varepsilon^2\,[F_{20}(s) + F_{01}(s)N] + \ldots \tag{3.3}$$

This equation must represent f for large N and small n (the matching conditions).

It follows from (3.3) that the inner expansion has the form (3.1) and

$$f_{0N}, \qquad f_{1N} \to 0, \qquad N \to \pm\infty \tag{3.4}$$

We note that just as for an incompressible fluid [3] the solution of the Navier-Stokes equations, as is shown by consideration of special examples, consists of two parts, one represented asymptotically by a power series in ε and the other having exponential character with respect to ε^{-1}. Therefore the conditions (3.4) may be written more accurately for large N in the form $f_0(s,N) \sim F_{00}(s)$ + exponential terms, $f_1(s,N) \sim F_{10}(s)$ + exponential terms, $f_2(s,N) \sim F_{20}(s) + F_{01}(s)N$ + exponential terms, and so on. (The functions $F_{kl}(s)$ are different for $N \to \pm\infty$.)

4. We derive the equations for the coefficients F_1 in region 1 (Fig.1). Here it is convenient to use Cartesian coordinates x, y, z. The expansion is taken in the form

$$\begin{aligned} p &= p_0(x, y, z) + \varepsilon p_1(x, y, z) + \ldots, \qquad & T &= T_0(x, y, z) + \varepsilon T_1(x, y, z) + \ldots \\ \rho &= \rho_0(x, y, z) + \varepsilon\rho_1(x, y, z) + \ldots, \qquad & \mathbf{v} &= \mathbf{v}_0(x, y, z) + \varepsilon\mathbf{v}_1(x, y, z) + \ldots \end{aligned} \tag{4.1}$$

Here $\mathbf{v}(v_x, v_y, v_z)$ is the velocity vector. Substituting (4.1) into the equations of continuity, momentum and energy, and taking into account the fact that the viscous terms do not affect the terms in ε, while the terms with subscript zero are constant, corresponding to the uniform stream at $x = -\infty$, we obtain

$$\frac{\partial v_{1x}}{\partial x} = -\frac{\partial p_1}{\partial x}, \qquad \frac{\partial v_{1y}}{\partial x} = -\frac{\partial p_1}{\partial y}, \qquad \frac{\partial v_{1z}}{\partial x} = -\frac{\partial p_1}{\partial z} \tag{4.2}$$

$$\frac{\partial}{\partial x}\left(\frac{p_1}{p_0} - \gamma\frac{\rho_1}{\rho_0}\right) = 0, \qquad \frac{\partial \rho_1}{\partial x} + \frac{\partial v_{1x}}{\partial x} + \frac{\partial v_{1y}}{\partial y} + \frac{\partial v_{1z}}{\partial z} = 0$$

We require that

$$\mathbf{v}_1,\ p_1,\ \rho_1 \to 0 \quad \text{as} \quad x \to -\infty \tag{4.3}$$

in any direction different from the characteristic one (see Section 7).

From (4.2) and (4.3) we obtain

$$p_1 = -v_{1x}, \qquad \gamma\frac{\rho_1}{\rho_0} = \frac{p_1}{p_0}, \qquad \frac{\partial v_{1y}}{\partial x} - \frac{\partial v_{1x}}{\partial y} = \frac{\partial v_{1z}}{\partial x} - \frac{\partial v_{1x}}{\partial z} = 0 \tag{4.4}$$

that is, in the plane and axisymmetric cases curl $\mathbf{v}_1 = 0$ and $\mathbf{v}_1 = \operatorname{grad}\varphi$, where φ satisfies Equation

$$-m^2\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0, \qquad m = (M_\infty^2 - 1)^{1/2} \tag{4.5}$$

We now show that the condition (4.3) imposes no limitations on $\mathbf{v}_1$ on a finite portion of the arc CBC' (Fig.1). This is most easily obtained for the plane case. Here $\partial/\partial z = 0$ and the general solution of (4.5) is $\varphi = g_1(x + my) + g_2(x - my)$, where g_1 and g_2 are determined if $\mathbf{v}_1$ is given on CBC' (Fig.1).

The condition (4.3) is satisfied if $g_1'(\pm\infty) = g_2'(\pm\infty) = 0$. The prime indicates differentiation. The same result is obtained also for the axisymmetric case if the solution of (4.5) is represented by the well-known formula of Volterra.

Consequently the condition of decay of disturbances in region 1 as $x \to -\infty$ (Fig.1) leaves $u_1(s,n)$ and $v_1(s,n)$ arbitrary on the finite segment CBC', whereas p_1 and ρ_1 are related by the condition (4.4), which we write in the form

$$p_1 / p_0 = \gamma\rho_1 / \rho_0, \qquad p_1 = -(u_1\cos\theta + v_1\sin\theta) \tag{4.6}$$

5. We consider the flow inside the shock wave (region 2 of Fig.1). The expansions for the solution are taken in the form (3.1)

$$\begin{aligned} p &= p_0(s,N) + \varepsilon p_1(s,N) + \ldots, \qquad & T &= T_0(s,N) + \varepsilon T_1(s,N) + \ldots \\ \rho &= \rho_0(s,N) + \varepsilon \rho_1(s,N) + \ldots, \qquad & u &= u_0(s,N) + \varepsilon u_1(s,N) + \ldots \\ v &= v_0(s,N) + \varepsilon v_1(s,N) + \ldots, \qquad & N &= n\varepsilon^{-2} \end{aligned} \tag{5.1}$$

Transforming to N and s in Equations (1.1) to (1.4) we obtain the system

$$\begin{gathered} (\rho v)_N = O(\varepsilon^2), \qquad \rho v u_N = (\mu u_N)_N + O(\varepsilon^2) \\ \rho v v_N + p_N = [(\lambda + 2\mu)v_N]_N + O(\varepsilon^2) \\ \rho v T_N - v p_N = -\sigma^{-1}(\mu T_N)_N + (\lambda + 2\mu)v_N^2 + \mu u_N^2 + O(\varepsilon^2) \\ p = [(\gamma - 1)/\gamma]\rho T \end{gathered} \tag{5.2}$$

The curvature of the shock wave affects only terms of order ε^2 in (5.2), so that the equations for the coefficients with subscripts 0 and 1 are obtained just as in the one-dimensiomal case, if $u_N = 0$. We give only the equations from which it follows that $u_{0N} = u_{1N} = 0$. From the relations

$$\rho_0 v_0 = a = \text{const}, \qquad a u_{0N} = (\mu_0 u_{0N})_N, \qquad \mu_0 = \mu_0(T_0)$$

it follows that

$$\mu_0 u_{0N} = \text{const} \exp\left(\int \frac{a}{\mu_0} dN\right)$$

Since $0 \neq \mu_0 < \infty$, then $u_{0N} \to 0$ as $N \to \pm\infty$ only in the case $u_{0N} \equiv 0$. For u_{1N}, using $u_{0N} = 0$, we obtain $a u_{1N} = (\mu_0 u_{1N})_N$; hence under the condition that $u_{1N} \to 0$ as $N \to \pm\infty$ it follows that $u_{1N} = 0$

We introduce for convenience the symbol $\{f\} = (f)_{N\to+\infty} - (f)_{N\to-\infty}$. Then for the one-dimensional problem there exist the well-known relations

$$\{\rho v\} = 0, \qquad \{\rho v^2 + p\} = 0, \qquad \left\{\frac{\gamma}{\gamma-1}\frac{p}{\rho} + \frac{v^2}{2}\right\} = 0 \tag{5.3}$$

which follow simply from the conditions

$$v_N, \quad p_N \quad \rho_N, \quad T_N \to 0, \quad N \to \pm\infty$$

We may therefore expect that the relations connecting p_0, p_1, ρ_0, ρ_1, u_0, u_1, v_0 and v_1 as $N \to \pm\infty$ will be obtained from (5.3) upon substituting (5.1) and equating coefficients of like powers of ε. They have the form

$$\begin{aligned} &\{u_0\} = 0, \quad \{p_0 v_0^2 + p_0\} = 0, \quad \left\{\frac{\gamma}{\gamma-1}\frac{p_0}{\rho_0} + \frac{v_0^2}{2}\right\} = 0 \quad \{\rho_0 v_0\} = 0 \\ &\{u_1\} = 0, \quad \{2v_0\rho_0 v_1 + v_0^2\rho_1 + p_1\} = 0, \quad \{\rho_0 v_1 + \rho_1 v_0\} = 0 \end{aligned} \tag{5.4}$$

$$\left\{\frac{\gamma}{\gamma-1}\left(\frac{p_1}{\rho_0} - \frac{p_0\rho_1}{\rho_0^2}\right) + v_0 v_1\right\} = 0 \tag{5.5}$$

The author has also obtained (5.4) and (5.5) directly from the equations for the quantities with subscripts 0 and 1 and the conditions that the derivatives of those quantities tend to zero as $N \to \pm\infty$ (see (3.4)). The relations (5.4) are the usual conditions on a shock wave for an inviscid stream. In the relations (5.5) the quantities u_1 and v_1 are arbitrary as $N \to -\infty$, but p_1 and ρ_1 are determined in terms of them by means of (4.6). If we eliminate u_1 and v_1 from (5.5) as $N \to -\infty$, we obtain two relations connecting u_1, v_1, p_1 and ρ_1 as $N \to +\infty$ $(n \to 0)$, that is, the desired relations on the shock wave.

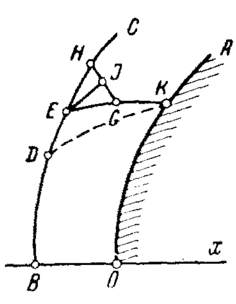

Fig. 3

Omitting the rather easy computations, we give the final result

$$\begin{aligned} &u_1 \cos\theta + v_0 v_1 + [\gamma / (\gamma - 1)] (p_1 \rho_0^{-1} - p_0 \rho_0^{-2} \rho_1) = 0 \\ &u_1 \cos\theta - \rho_0 \sin\theta v_1 + (v_0^2 - v_0 \sin\theta)\rho_1 + p_1 = 0 \end{aligned} \tag{5.6}$$

All quantities are taken on approaching CBC' from the side of region 3 (Fig. 1).

6. The equations for the coefficients with subscript 1 in Expansions (2.1) in region 3 (Fig.1) are actually obtained from the equations of inviscid flow (the viscous terms being of order ε^2) and have the form

$$[(r + n\cos\theta)^j (\rho_0 u_1 + \rho_1 u_0)]_s + [(1 + kn)(r + n\cos\theta)^j (\rho_0 v_1 + \rho_1 v_0)]_n = 0$$

$$u_0 u_{1s} + u_1 u_{0s})(1 + kn)^{-1} + v_0 u_{1n} + v_1 u_{0n} + k(1 + kn)^{-1}(u_0 v_1 + u_1 v_0) + \\ + (1 + kn)^{-1}(\rho_0^{-1} p_{1s} - \rho_1 \rho_0^{-2} p_{0s}) = 0$$

$$(u_0 v_{1s} + u_{0s} v_1)(1 + kn)^{-1} + v_0 v_{1n} + v_1 v_{0n} - k(1 + kn)^{-1} 2u_0 u_1 + \\ + p_{1n}\rho_0^{-1} - \rho_1 \rho_0^{-2} p_{0n} = 0$$

$$\rho_0 [(u_0 T_{1s} + u_1 T_{0s})(1 + kn)^{-1} + v_0 T_{1n} + v_1 T_{0n}] + \rho_1 [u_0 T_{0s}(1 + kn)^{-1} + \\ + v_0 T_{0n}] - (u_0 p_{1s} + u_1 p_{0s})(1 + kn)^{-1} - v_0 p_{1n} - v_1 p_{0n} = 0$$

$$p_1 = (\rho_0 T_1 + \rho_1 T_0)(\gamma - 1)/\gamma \qquad (6.1)$$

The type of Equations (6.1) is determined by the solution of the inviscid problem. In particular, the characteristics of (6.1) coincide with the characteristics of the inviscid problem.

The transonic region (Fig.3) in which u_1, v_1, p_1, ρ_1 and T_1 must first be found is bounded by the limiting characteristic EK, the surface of the body OK, the axis of symmetry BO, and the segment BE of the shock wave (DK being the sonic line). It is necessary to find the solution of (6.1) with the following boundary conditions: the normal component of $\mathbf{v}_1$ is given on OK [2]; the condition of symmetry on BO; the two conditions (5.6) on the arc EB, whose location is known from the solution of the inviscid problem; and on EK, whose location is also known, a relation between the differentials of the unknown quantities appearing in the characteristics of Equations (6.1) (boundedness of the derivatives on the characteristic EK).

The question of existence and uniqueness for such a problem requires further consideration. In the supersonic region $CEKA$ the answer to that question can be obtained by means of an investigation of the possibility of constructing the flow by the numerical method of characteristics. Assuming that u_1, v_1, p_1 ρ_1 and T_1 are known on EK, we calculate the flow in the region $CEKA$. The essential step in the calculation is the determination of p_1, ρ_1, T_1 u_1 and v_1 at point H (Fig.3) from the data at points E and G. Along a streamline – the characteristic EI of Equations (6.1), we may find dS_1, where $S_1 = p_1 p_0^{-1} - \gamma \rho_1 \rho_0^{-1}$. Replacing the differential by a finite increment, we find the value of S_1 at point I from S_1 at point E, and from values of S_1 at points I and G we find S_1 at point H by linear extrapolation. From the relations between dp_1, $d\rho_1$, du_1 and dv_1 along the characteristic HG of Equations (6.1) we obtain still another condition on u_1, v_1, p_1 and ρ_1 at point H. Then adding the conditions (5.6) we obtain a system of four independent linear algebraic equations for determining u_1, ρ_1, v_1 and p_1 at point H.

The correctness of the flow calculation in the supersonic region allows us to hope that the boundary-value problem considered above for the transonic region is also correct.

7. In the inviscid problem there springs from a point on the contour where the curvature is discontinuous a characteristic L, along which the derivatices of u, v, p, ρ and T, with respect to n suffer discontinuities (n now being measured normal to L and s along L). If viscosity is considered, there exists, just as in the case of a shock wave, a transition region outside of which expansions of the form (2.1) are assumed to be valid. The terms of order ε in these expansions may evidently suffer discontinuities on L. From the point of intersection of L with the shock wave CBC' (Fig.1) two characteristics emerge into region 1, for which $dy/dx = \pm m^{-1}$, with $m = (M_\infty^2 - 1)^{1/2}$, and near which there exists transition region outside of which the solution is represented by expansions of the form (2.1), and inside by expansions of the form

$$f = f_0 + \varepsilon f_1 (\eta, s) + \ldots, \qquad f_0 = \text{const}, \qquad \eta = n\varepsilon^{-1} \tag{7.1}$$

Let us use the symbols $\{f\} = (f)_{\eta\to+\infty} - (f)_{\eta\to-\infty}$; for the coefficients of the expansion (7.1) there are the relations

$$\{p_0\} = \{\rho_0\} = \{u_0\} = \{v_0\} = 0, \quad \{p_1\} = v_0^2 \{\rho_1\}, \qquad \{v_1\} \rho_0 + \{\rho_1\} v_0 = 0, \qquad \{u_1\} = 0$$

Hence it follows that

$$\{p_1 + v_{1x}\} = \{p_1 p_0^{-1} - \gamma\rho_1\rho_0^{-1}\} = 0$$

that is, it is found that the Formulas (4.4) are valid in the presence of discontinuities in the terms of order ε in region 1 (Fig.1).

As $x \to -\infty$ the transition regions near the characteristics determined by Equations $dy/dx = \pm m^{-1}$ broaden, so that the characteristic directions must be excluded in conditions (4.3).

8. In conclusion we note that the method of inner and outer expansions was applied by Germain [4] to obtain conditions on the shock wave, but he expanded the inner solution in powers of ε^2 and so missed the terms of order ε that are of interest in the present problem.

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